* Review 2st week
* Random Variable
* **Probability Distribution function**
* Prob. Density Function

Then is **the probability density function**,

* Properties
* Random vectors (X,Y)
* Joint Probability Distribution, density
* Marginal Probability Distribution, density
* and are called **independent if**

2. Or

* Expectations and Moments of a Random Variable
* The mean:
* The sample mean (if is identically independent distribution random variables)

If

Then the mean of the sample mean is

1. The square mean / second moment
2. The higher order moment
3. The variance
4. The standard deviation
5. The sample variance

This is a random variable. And the **unbiased** estimator of

* Unbiased estimator / biased estimator

If , then C is the unbiased estimator, otherwise the biased estimator

* The minimum variance estimator /the least square error estimator
* Week\_3
* Definition : The Covariance (p.44)

The covariance of and , is

Def : Two R.V. are **uncorrelated** if

%% Kim’s comment

* If you shout “Hi” in the canal, the echo ‘Hi” is correlated to your voice, but the echo “ Good morning” is not correlated to “Hi”.
* If are independent, then they are uncorrelated, but they may not be independent %%

%% Random Vector covariance

* Fact: Two Gaussian random variables are uncorrelated if is a diagonal matrix
* **Prop. 2.29(page 50) Uncorrelated Gaussian random variables are independent**

Proof**:**  are Gaussian, then

Since they are uncorrelated,

Substituting this yields

**QED**

* Theorem 2.30. If is a Gaussian random vector with mean , and covariance, , and if , where is a Gaussian random vector with zero mean and covariance, , then is a Gaussian random vector with mean, , and covariance, .
* **Theorem 2.30 : A R.V , another R.V. and they are independent**. The mean and covariance of are

Proof: Let’s apply the basic definition.

The Covariance is

The first term

Hence

%%% In the text book, Theorem 2.30, **Independence should be assumed** %%%

* The covariance of a uncorrelated (so independent) Gaussian is a diagonal matrix,

%%% Kim’s comment: Linear matrix theory: similar transform

For any semi-positive symmetric matrix , there is a **similar transform matrix**such that

Hence the covariance for any gaussian Random vectors (correlated), there exits a such that

* Any Gaussian Random vectors, we can find a transformed Random Vectors which is uncorrelated(independent).
* Independency is important to calculate the probability. You know the Gaussian probability table, but it is a scalar. So it you want to calculate the joint probability which may be correlated, first find a similar transform matrix to generate a diagonal covariance matrix. Then you may calculate the joint probability as a separate probability.

%%%

* **The central limit theorem**

Theorem 2.31. Let be i.i.d. random variables with finite mean and variance,

and denote their sum as . Then the distribution of the normalized sum

is a Gaussian distribution with mean 0 and variance 1 in the limit as

* Proof : textbook P.52
* Remarks:

1. See, the condition, that means   
   the mean and the variance is constant, but the experiment is many time processing. For example,
2. A die, which is fair or not, you roll the same die many times. Then the mean of the sum () is a Gaussian if .
3. Some RV has no mean, then it will not be applicable.
   1. Conditional Expectations and Conditional Probabilities

* The conditional expectation
* Remarks
* is a constant, means it is not random variable.
* if is a constant, then is a constant
* if is a RV, then is a **Random Variable** of y
* **Iterated expectation(See the proof at p.57 and remember)**

%%% Kim’s comment

Even if we do not know .

I should say, this formula cannot emphasize too much! This very simple fact use diverse applications, big data, machine learning, and dynamic system analysis. We should **remember** this.

%%%

* Lemma 2.34.
  1. Stochastic Process
* Def. 2.36. A stochastic process is a family of random variables, , indexed by a real parameter and defined on a common probability space .

%%% Kim’s comment

A stochastic process (or random process) is a time varying random variable, i.e., for any fixed , the process is a random variable.

%%%

* Ex. 2.37
* Def. 2.38.

1. A stochastic process is said to be continuous in probability at t if

for all

1. Skip: A stochastic process is said to be separable if there exists a countable, dense set such that for any closed set

differ by a set such that

* Skip: Theorem 2.40. The rational numbers in provide a separating set S.
* Def. 2.42. Let X be a random process defined on the time interval, T. Let

be a partition of the time interval, T. If the increments,are mutually independent for any partition of T, then X is said to be a process with **independent increments**.

* Def. 2.43 We say that a random process, X, is a Gaussian process if for every finite collection, the corresponding density function,

is a Gaussian density function.

* Def. 2.44 We say that a random process X is a Gaussian process if every finite linear combination of the form

is a Gaussian random variable

* Def 2.45. A random process, where T is a subset of the real line, is said to be a **Markov process** if for any increasing collection

or, equivalently

* 1. Gauss-Markov Processes–**The fundamental**

1. Dynamics

* State , is a known matrix, is a Gaussian Random sequence.

1. Given Conditions
2. Noise

where

1. The states
2. The correlation

which implies

1. The mean and covariance

* The mean
* The covariance
  1. Non-linear Stochastic Difference Equations 🡪 skip